Electrical Resistivity Tomography

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Outline

- ERT and 1-D ERT inverse problem
- Newton-Raphson method and regularizations
- Objective function landscape
- Global search method

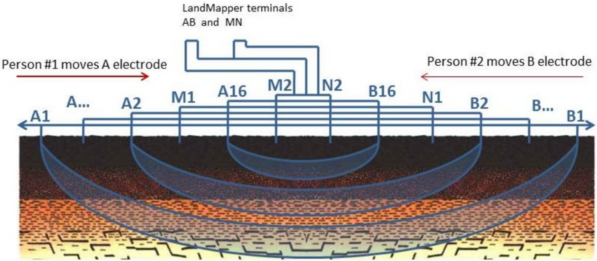
Electrical Resistivity Tomography

- Direct current pumped into Earth
- Voltage measured at reference electrodes
- Tikhonov famously used this method in the 1940s

Ekrem Canli, Wikipedia

Schlumberger Vertical Sounding Method

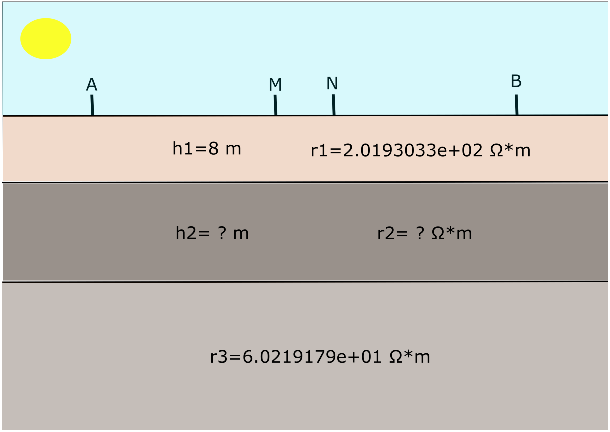
- Feeding electrodes out wide
- Reference electrodes in middle
- Feeding electrodes moved out to image deeper layers



Landviser LLC

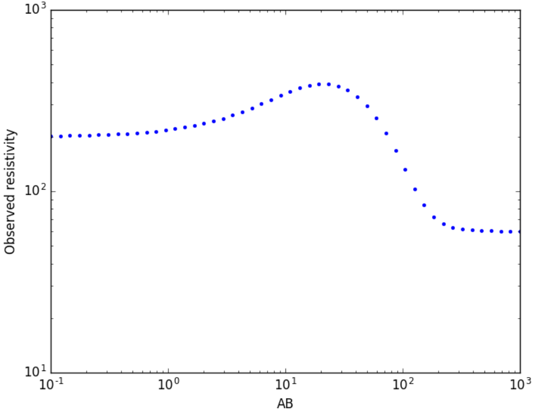
Resistivity Problem

- 2 layers and a half-space
- Properties of second layer unkown



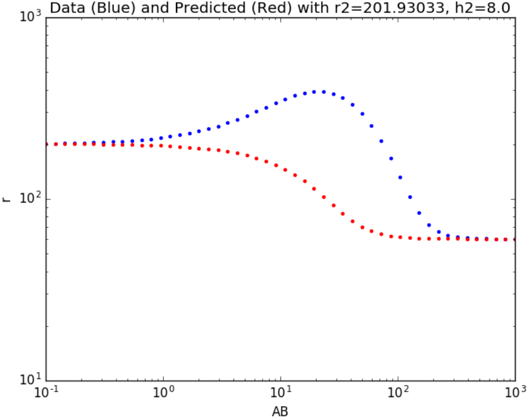
Resistivity Data

- 50 data points
- Small AB is top layer
- Large AB is bottom half-space



Resistivity Model

- $\mathbf{d} = \mathbf{g}(\mathbf{m})$
- $\mathbf{m} = [r_2, h_2]^T$
- High nonlinearity of model makes directed searches difficult



Newton-Raphson Method

Solve a series of linearized least-squares inversion problems for corrections to an initial guess for model parameters $\mathbf{m_0} = [r_2^0, h_2^0]^T$

$$\mathbf{res} = \mathbf{G}\,\mathbf{dm}$$

where $\mathbf{res} = \mathbf{d} - \mathbf{g}(\mathbf{m_0})$, $\mathbf{dm} = \mathbf{m} - \mathbf{m_0}$, and the columns of $\mathbf{G}$ are the first-partials of $\mathbf{g}$ with respect to each model parameter. Then

$$\mathbf{dm} = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{res}$$

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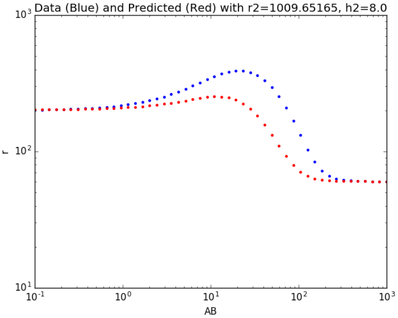
Note: derivatives are approximated with finite differences.

$$\frac{\partial \mathbf{g}(r_2^0, h_2^0)}{\partial r_2} \approx \frac{\mathbf{g}(r_2^0 + \Delta r_2, h_2^0) - \mathbf{g}(r_2^0, h_2^0)}{\Delta r_2}$$

Newton-Raphson Iteration

Exit when $||\mathbf{dm}|| < \text{tol}$

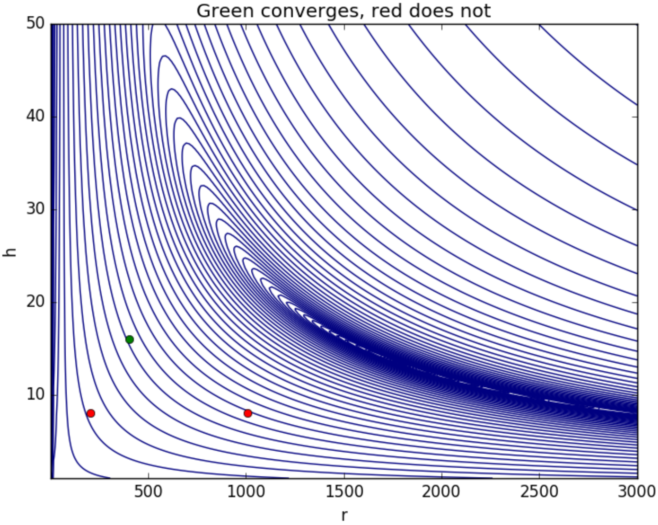


Does not converge

Converges, final residual$= 0.03813$

Objective Function Landscape

Objective function $(\mathbf{d} - \mathbf{g}(\mathbf{m}))^T(\mathbf{d} - \mathbf{g}(\mathbf{m}))$



Newton-Raphson Iteration Visualized

Newton-Raphson with Regularization

Can regularize by padding our linear problems with two more equations

$$\begin{pmatrix} \mathbf{G} \\ \epsilon \mathbf{R} \end{pmatrix} \mathbf{dm} = \begin{pmatrix} \mathbf{res} \\ \mathbf{0} \end{pmatrix}.$$

Then

$$\mathbf{dm} = \left(\mathbf{G}^T \mathbf{G} + \epsilon^2 \mathbf{R}^T \mathbf{R}\right)^{-1} \mathbf{G}^T \mathbf{res}.$$

Damped Regularization Visualized

Weighting Regularization

- Damping is not advantageous as it equally penalizes changes in resistivity and thickness
- $r_2 \gg h_2$
- Solution is to weight the damping to correct for this

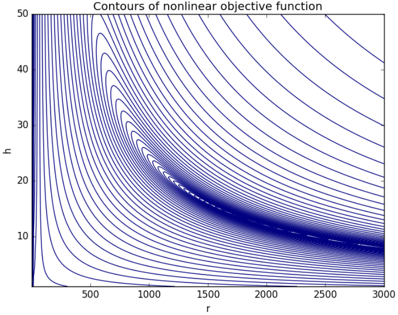
$$\mathbf{R} = \begin{bmatrix} \frac{1}{r_2^0} & 0 \\ 0 & \frac{1}{h_2^0} \end{bmatrix}$$

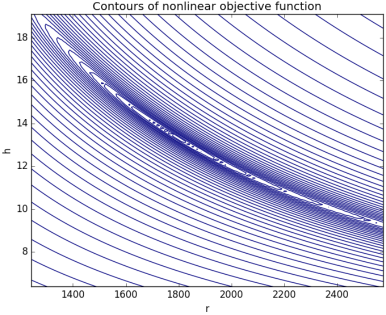
Weighting Regularization Visualized

Global Search Methods

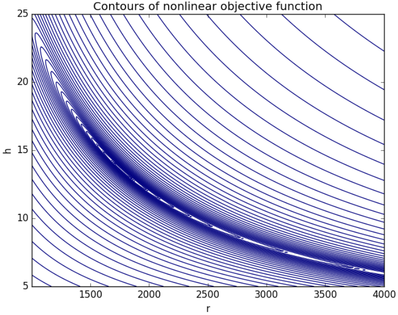
- Even improved directed search methods are imperfect for this problem
- Global search needed to accurately demonstrate location of minimum
- Grid search rather than stochastic method used

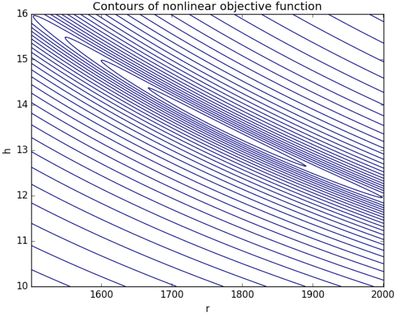
Objective Function Landscape



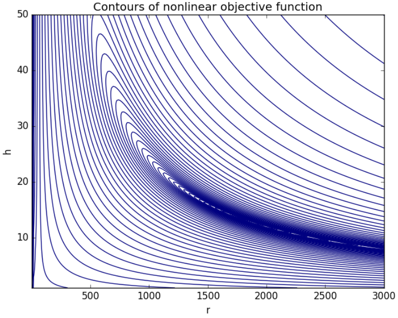


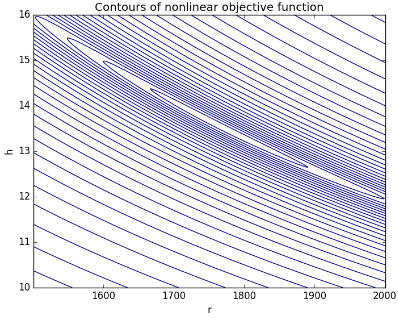
Objective Function Landscape contd.





Global Grid Search





- 400 by 200 data points
- Minimum at $r_2 = 1749.744$, $h_2 = 13.649$
- Minimum residual of 0.03874
- Approx 50 min. computation time

- 200 by 250 data points
- Minimum at $r_2 = 1766.060$, $h_2 = 13.558$
- Minimum residual of 0.03813
- Approx 35 min. computation time

Adaptive Grid Search and N-R Method

Iteratively search over coarse grids (10 by 10), refining around minimum point. Then use N-R when sufficiently close to minimum.

- Initial grid $1 < r_2 < 10000$ $\Omega{\cdot}$m by $0.1 < h2 < 200$m
- Converges after 10 total iterations to $r_2 = 1770.759$ $\Omega{\cdot}$m and $h_2 = 13.518$m
- Final residual is 0.03813
- Computation time 19s

Thanks!

Thanks for your attention! Any questions?